

Comment on “Experimental realization of Popper’s experiment: Violation of the uncertainty principle?”

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Abstract

Application of the uncertainty principle to conditional measurements is investigated, and found to be valid for measurements on separated sub-systems. In light of this, an apparent violation of the uncertainty principle obtained by Kim and Shih in their realization of Popper’s experiment [1] is explained through analogy with a simple optical system.

1 Introduction

The entangled photon pairs produced in spontaneous parametric down-conversion (SPDC) have been used in a range of experiments to demonstrate non-local correlations in quantum mechanics. In this paper we examine the results of one such experiment, in which Yoon-Ho Kim and Yanhua Shih obtain an apparent violation of the uncertainty principle [1].

Their experiment is the modern realization of a thought experiment created by Karl Popper in the early 1930’s to illustrate his doubts about the uncertainty principle [1][2][3]. Although his proposal was flawed because of its reliance on a stationary point source, which is itself forbidden by the uncertainty principle [4], Kim and Shih have recreated the essence of Popper’s experiment without this problem using SPDC and a converging lens. Surprisingly, their experimental data appears to show a violation of the uncertainty principle in agreement with Popper’s original prediction.

After a review of Kim and Shih’s experiment, we investigate the validity of the uncertainty principle when dealing with entangled systems and conditional measurements. In light of this, we then look more closely at their experiment and offer an alternative explanation for the results. As with Popper’s original experiment, we will show that the width of the source plays a crucial role.

2 The experiment

As explained in [1], Kim and Shih’s experiment is conceptually equivalent to the “unfolded” schematic given in Fig. 1, in which an SPDC source, lens and slits all lie on a common x -axis. The central converging lens (of focal length 500mm) is positioned 255mm from the SPDC source and 1000mm from each of two parallel slits (A and B), with slit B closest to the source. Pairs of entangled photons are generated at the source by an external pump-laser, and their trajectories are measured on either side of the slits by detectors D_1 and D_2 . A collecting lens is used to channel light passing through slit A into the fixed detector D_1 , while D_2 is scanned along the y -axis 500mm behind slit B. The results

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are then combined in a coincidence circuit to yield a conditional measurement: *The y coordinate of photon 2 in the plane of D_2 given that photon 1 passes through slit A and is detected at D_1 .* Two cases are studied; (i) in which slit B is the same width as slit A (0.16mm), and (ii), in which slit B is wide open.

Although the photons are created with a large uncertainty in position and momentum there are strong correlations between each pair due to the phase matching conditions of SPDC. In this “unfolded” schematic the momenta of the photons in each pair ($\hbar\mathbf{k}_1$ and $\hbar\mathbf{k}_2$)¹ are almost precisely anti-correlated, with $\mathbf{k}_1 + \mathbf{k}_2 \simeq 0$. Because of this, the two-photon trajectories are well represented by straight lines and may be treated like optical rays².

We can construct a single-photon system which will generate approximately the same results as this experiment by replacing detector D_1 with a lamp and removing the SPDC source, as shown in Fig. 2. Individual photons then propagate from the source at D_1 through the collection lens, slit A, the central lens, and slit B before being detected at D_2 . As discussed in [6] and [7], results for this simple optical setup will be very similar to coincidence measurements in the SPDC experiment. In particular, we expect a “ghost image” of slit A to be observed in coincidence measurements on the photon pair, just as a conventional image is visible in the single-photon setup, and this has been verified experimentally by Pittman *et al.* [8].

By positioning both slits two focal lengths (1000mm) away from the lens, we can produce an unmagnified image of slit A in the plane of slit B³. If photon 1 passes through slit A and is detected at D_1 then photon 2 must pass through the image in the plane of slit B, as shown in fig. 1. The image slit should be the same width as slit A (0.16mm), so one would not expect the behaviour of photon 2 to be affected if slit B is also narrowed to this width. However, according to Kim and Shih’s results *the momentum spread of photon 2 when slit B is narrowed is almost three times that when slit B is wide open*. It appears that the presence of a physical slit affects the results even though it does not change the spatial confinement of the photon.

The momentum uncertainty of photon 2 in the image plane can be deduced approximately from the width of its spatial distribution at D_2 . In case (i), with slit B narrowed to the same width as slit A (0.16mm), the distribution at D_2 is 4.4mm wide. However, when slit B is wide open in case (ii) the width at D_2 is reduced to 1.6mm. Using simple geometrical arguments we obtain $\Delta_{(ii)}p_y \approx 0.36\Delta_{(i)}p_y$, where $\Delta_{(i,ii)}$ refers to the uncertainty in cases (i) and (ii) respectively. If we accept the above arguments and take the uncertainty in y to be the same in both cases ($\Delta_{(i)}y = \Delta_{(ii)}y = 0.16\text{mm}$) then there is significant reduction in $\Delta y \Delta p_y$, which suggests a violation of the uncertainty principle⁴.

3 Conditional measurements and the uncertainty principle.

The uncertainty principle constrains the results of any measurement of non-commuting observables, and can be represented by the general inequality [5]

$$\Delta_\psi A \Delta_\psi B \geq \frac{1}{2} |\langle \psi | [\hat{A}, \hat{B}] | \psi \rangle|, \quad (1)$$

which relates the standard deviations of observables \hat{A} and \hat{B} . If $|\psi\rangle$ describes a system of several particles we can construct an inequality for the position \hat{x}_n and momentum \hat{p}_n of the n ’th particle,

¹Photons 1 and 2 are often referred to as ‘signal’ and ‘idler’ photons respectively.

²This property is the major motivation for “unfolding” the experiment in this way, the procedure for which is explained more fully in refs. [1] and [8]

³This follows from the Gaussian thin lens equation $\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$, where a and b are the positions of slit A and its image relative to the lens, and f is the focal length. $a = b = 2f$ gives an unmagnified solution.

⁴It is difficult to obtain a definite violation of the uncertainty principle because we are looking at peak widths rather than standard deviations. The *sinc*² function generated by diffraction actually has an infinite standard deviation, and alternative measures of uncertainty and uncertainty relations are therefore required [9].

where $[\hat{x}_n, \hat{p}_n] = i\hbar$. This yields a familiar Heisenberg uncertainty relation for each particle in the system, regardless of any entanglement between them,

$$\Delta x_n \Delta p_n \geq \frac{\hbar}{2} \quad \forall n. \quad (2)$$

However, many experiments on entangled systems actually probe conditional behaviour rather than single-particle properties. In Kim and Shih's experiment the crucial quantities are '*the position and momentum of photon 2 in the plane of slit B given that photon 1 is detected at D_1* '. Can we still apply the uncertainty principle to such conditional quantities?

To investigate this, we will consider a more general case: '*A measurement M_2 of one of two non-commuting observables \hat{A} or \hat{B} given that a measurement M_1 of \hat{O} obtains the result o* '.

If M_1 precedes M_2 , the situation is simple. First, measurement M_1 acts on $|\psi\rangle$ according to the projection postulate, giving

$$|\psi'\rangle = \frac{\hat{P}_o |\psi\rangle}{(\langle\psi|\hat{P}_o|\psi\rangle)^{\frac{1}{2}}}, \quad (3)$$

where \hat{P}_o is an operator which projects the system onto the eigenstate(s) associated with result o . The state will then undergo some unitary evolution into $|\psi''\rangle = \hat{U}|\psi'\rangle$ before measurement M_2 . As seen above, the uncertainty principle is applicable to measurements on any quantum state, and will therefore constrain the results obtained in measurement M_2 in the normal way, with

$$\Delta A \Delta B \geq \frac{1}{2} |\langle\psi''|[\hat{A}, \hat{B}]|\psi''\rangle|. \quad (4)$$

If M_1 occurs after M_2 the situation is more complex, as the state on which M_1 acts will depend on which observable \hat{A} or \hat{B} was measured in M_2 and the result which was obtained. However, if $|\psi\rangle$ represents a bi-partite system in which both sub-systems evolve independently, and M_1 and M_2 act on different sub-systems, then the results of M_2 will be bound by the uncertainty relation regardless of the time-ordering of M_1 and M_2 .

In such cases, the evolution of sub-system 1 is decoupled from that of sub-system 2 and can be described by the operator $\hat{P}_o \hat{U}_1(\tau)$, where \hat{U}_1 is the unitary time evolution operator for sub-system 1, and τ is a time interval containing measurement M_2 . By tracing \hat{P}_o backwards in time we can rewrite this evolution as $\hat{U}_1(\tau) \hat{P}'_o$, where $\hat{P}'_o = \hat{U}_1^\dagger(\tau) \hat{P}_o \hat{U}_1(\tau)$ represents an equivalent projection before temporal evolution. A measurement M_1 which obtains the result o *after* measurement M_2 can therefore be replaced by a measurement M'_1 obtaining o' (with corresponding projector \hat{P}'_o) *before* measurement M_2 . Because measurements on independent subsystems can always be shifted through time in this way, their ordering becomes irrelevant. We can always recast M_1 as a measurement preceding M_2 , and use the above arguments to apply the uncertainty principle to the results.

Any conditional measurement in which M_1 and M_2 act on separately evolving subsystems will therefore be bound by the uncertainty principle. In Kim and Shih's experiment these subsystems are the two photons created during SPDC, which do not interact with each other after their initial creation and must evolve independently between measurements when they are space-like separated. We are specifically interested in a measurement M_2 of the position $\hat{A} = \hat{y}$ or momentum $\hat{B} = \hat{p}_y$ of photon 2 in the plane of slit B given that a measurement M_1 on photon 1 detects it at D_1 . According to equation (4) and the commutation relation $[\hat{y}, \hat{p}_y] = i\hbar$, we expect photon 2 to obey the standard Heisenberg relation $\Delta y \Delta p_y > \hbar/2$ even though y and p_y refer to conditional quantities, yet the results of the experiment appear to violate this relation.

4 An explanation of the results

The measured value of Δp_y for the conditionally localised photon is approximately one third that for diffraction at a physical slit. To obey the uncertainty relation derived above, we would expect Δy to be 2-3 times larger than the slit width to compensate for its reduced momentum spread. However, we know that the photon is confined to the unmagnified image of the slit, so Δy should equal the slit width. How can we resolve this apparent paradox?

The answer lies in our assumption that the image is perfect. If we consider the blurring introduced by the finite width of the SPDC source, we find that the image is actually 2-3 times larger than the physical slit, precisely as predicted by the uncertainty relation. Narrowing slit B has a noticeable effect because it picks out the centre of the blurred image (where a perfect image would lie), and thus alters the spatial confinement of the photon.

The primary factor limiting image resolution is the width of the region in which photon pairs are created, given by the diameter of the laser beam pumping the SPDC source ($\approx 3\text{mm}$). All photon trajectories must pass through the laser-pumped region because this is where photons are created during SPDC, and we should account for this in our single-photon model. By replacing the SPDC source with a 3mm aperture only trajectories which pass through the source region will be carried over to the single-photon picture, and trajectories which lie outside the source region (and therefore do not correspond to a valid two-photon trajectory) will be eliminated. As we will see below, this intuitive step leads to a blurring of the image which explains the results without violating the uncertainty relation.

In terms of the SPDC process, we can understand the same effect as a result of imperfect phase matching. The accuracy of the phase matching condition $\mathbf{k}_1 + \mathbf{k}_2 \simeq 0$ is limited by the uncertainty relations

$$\Delta(k_{1y} + k_{2y})\Delta y_1 \geq \frac{1}{2}, \quad (5)$$

$$\Delta(k_{1y} + k_{2y})\Delta y_2 \geq \frac{1}{2}, \quad (6)$$

derived from equation (1). As Δy_1 and Δy_2 are limited by the source width, the momenta of the photons in each pair cannot be precisely anti-correlated, and representing their trajectories as straight lines through the source is only an approximation. It is by tracing these straight lines that we obtain a perfect image, so by disturbing them the uncertainty tends to cause blurring.

To understand the connection between the two approaches we equate $k_y = -k_{1y}$ and $k'_y = k_{2y}$ with the wavevector of a photon entering and leaving the source region respectively. Taking $\Delta y_1 = \Delta y_2 = \Delta y_s$ as a measure of the source width, equation (5) then becomes $\Delta(k'_y - k_y)\Delta y_s \geq \frac{1}{2}$, which indicates a diffraction-like disturbance at the source. A photon with a given incident wavevector k_y will acquire a momentum spread $\Delta k'_y$ on passing through the source which is characteristic of diffraction at a slit of width Δy_s and is governed by the uncertainty relation $\Delta k'_y \Delta y_s \geq \frac{1}{2}$. By replacing the SPDC source with an appropriate slit in the single-photon system we are therefore simulating the effect of imperfect phase matching. The fact that both the single-photon and phase-matching approaches yield the same predictions highlights the power and universality of the uncertainty principle.

We can estimate the width of the blurred image by treating the SPDC source as a rectangular aperture in the single-photon system. Using this simple model each point in the image is spread by convolution into a *sinc*² function of width (between first minima)

$$\Delta y = \frac{2D\lambda}{s} \quad (7)$$

where D is the distance from source to image (745mm), λ is the photon wavelength (702.2nm) and s is the source width (3mm). This gives a blurring of $\Delta y = 0.35\text{mm}$, which is more than double the

expected width of the image (0.16mm) and is of the right magnitude to account for the reduction in momentum spread observed in the results for case (ii). The accuracy of this analysis can be improved by using a slit profile which better represents the intensity of photon pair production in the SPDC source, but the underlying result remains the same; The narrower the source, the larger the blurring in the image and the greater Δy .

The corresponding reduction in Δp_y can be explained geometrically. For photons to travel from a source of width s through a point-like image a distance D ($\gg s$) away their trajectories must be bounded by the triangular region between the two, such that

$$\frac{\Delta p_y}{p} \simeq \frac{s}{D}. \quad (8)$$

When combined with (7) and the De Broglie relation $p = h/\lambda$ this gives the uncertainty relation $\Delta y \Delta p_y \approx 2h$ which is characteristic of single-slit diffraction at a rectangular aperture.

During the free evolution considered above, the momentum distribution of the photon is conserved. However, if slit B is narrowed to the same width as slit A (0.16mm) then only those photons passing through the centre of the blurred image will be detected at D_2 . This increased spatial confinement gives the photons a greater momentum spread and results in a broader pattern at D_2 , as observed in the results for case (i).

Interestingly, if D_1 detects all photons passing through slit A, then we will *only* obtain a *sinc*² diffraction pattern in case (i) if the image is blurred. Consider the wavefunction of the two-photon entangled system when photon 1 is in the jaws of slit A and photon 2 in the image⁵;

$$|\psi\rangle \propto \int_{-s/2}^{s/2} |y\rangle_1 | -y\rangle_2 dy. \quad (9)$$

The state of photon 2 is given by the reduced density matrix $\hat{\rho}_2 = \text{Tr}_1(|\psi\rangle\langle\psi|)$, which is actually an incoherent mixed state of the image points,

$$\hat{\rho}_2 \propto \int_{-s/2}^{s/2} |y\rangle_2 \langle y| dy. \quad (10)$$

Each point in the image will therefore evolve with an infinite momentum spread and over the tracking range of D_2 results will be almost constant. It is only when the image points are blurred into coherent functions which spread over the width of slit B that a *sinc*² interference pattern will be obtained in the results.

Some of the coherence may also be restored if D_1 only detects a subset of the photons passing through slit A. This is a form of quantum erasure, in which information about which part of slit A the photon passed through is lost.

5 conclusions

In their paper, Kim and Shih correctly claim that their experiment does not violate the uncertainty principle, but we disagree with their explanation. They insist that the photons propagating towards D_2 are conditionally localised to within $\Delta y = 0.16\text{mm}$ by the image of slit A, and that their reduced momentum spread in case (ii) is not a cause for concern because the uncertainty principle does not apply:

⁵These two events will not actually occur simultaneously, but as the photons evolve independently we can consider the wavefunction $|\psi\rangle = (\hat{U}_1(t_1) \otimes \hat{U}_2(t_2))|\psi_0\rangle$ where the two photons are effectively studied at different times

“A quantum must obey the uncertainty principle but the “conditional behaviour” of a quantum in an entangled two particle system is different. The uncertainty principle is not for conditional behaviour.”

In this paper, we have shown that the results of their experiment can be explained without this assertion. In fact, the uncertainty principle can be naturally extended to conditional measurements whenever they involve separately-evolving sub-systems, as is the case in their experiment.

As we have shown, blurring of the “image” not only explains the observed results without violating the uncertainty principle, but is actually *a necessary consequence* of the principle due to position-momentum uncertainty at the SPDC source. To explain the diffraction patterns observed we actually require blurring to introduce coherent superpositions across the slit width, and give a finite momentum spread. The effect is analogous to that which blurs point-source images (e.g. the image of a star) in any lens system with a finite aperture.

These conclusions are also supported by the results of the previous “ghost imaging” experiment of Pittman *et al.*, in which significant blurring is evident in the image.

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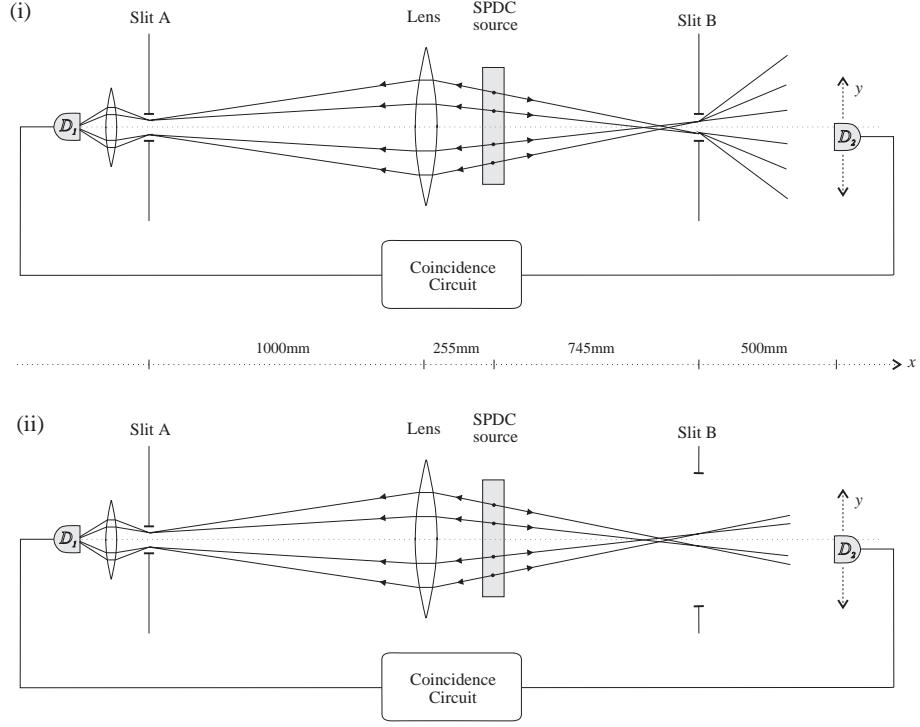


Figure 1: The “unfolded” schematic of Kim and Shih’s experiment, showing the two cases investigated. In case (i) slit B is narrowed to the same width as slit A (0.16mm), while in case (ii) slit B is wide open. Experimental results show that the momentum spread Δp_y of the photon passing through slit B is almost three times as large when the slit is narrowed.

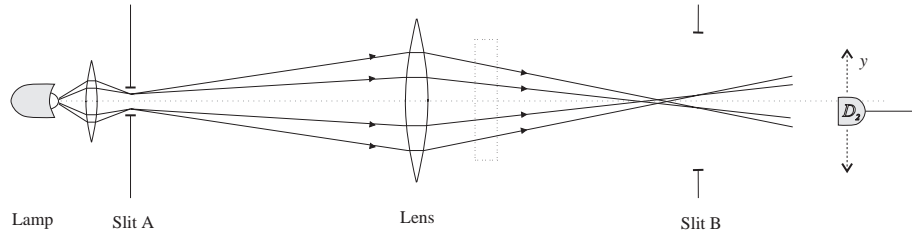


Figure 2: The schematic for an analogous single-photon experiment, in which D_1 is replaced by a lamp and the SPDC source removed. Note the similarity to figure 1 and the change in the direction of propagation of photons to the left of the lens.